**Data Analysis**

**4.1 Analysis Methodology**

With all required data collected the next step was analysis. In total 8 empirical tests of randomness were performed. These consisted of:

* The Chi-Squared Test
* The Kolmogorov-Smirnov Test
* The Serial Test
* The Gap Test
* The Poker Test
* The Runs Test
* The Serial Correlation Test
* The Birthday Spacings Test

Many of these tests would have been provided in the Dieharder test suite, including the Kolmogorov-Smirnov and Birthday Spacings tests, however after technical issues regarding the set up of a test battery the test suite shown above had to be produced manually within R Studio. The data analysis was completed in R primarily due to the facilities provided by the language for handling and visualising datasets as well as its wide array of test libraries that provided the functions necessary to produce the test suite shown above. Additional libraries such as ggplot2 and rjson also made R the obvious choice for analysis as the JSON datasets could easily be imported, processed, and graphed within R Studio. Although similar tools existed within languages like Python or MATLAB, the ggplot2 library available with R could produce much higher quality figures compared to Python alternatives like Matplotlib or Plotly while also providing a substantial amount of control to the user.

A screenshot of a computer code

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*Figure 43. The R analysis program importing libraries and JSON data*

**4.2 Chi-Squared Test of Dice and Coin Simulation Data**

The first test used in this investigation was the Chi-Squared test, which is an empirical test designed to calculate a V value from a sequence of random numbers and compare that value to a distribution table to determine the probability that such a sequence could be produced. The main caveat of the Chi-Squared test that limited its use to only dice and coin data is the distribution table which only considers sequences with up to 99 potential outputs. As all the pseudorandom sequences sampled in this investigation had a minimum of 100 potential outputs, it wasn’t feasible to apply Chi-Squared testing to them. However, for coin and dice data that had a maximum number of possible outputs of either 2 or 6, these datasets were easily mappable to the distribution table.

A math equation with numbers and symbols

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*Figure 44. The Chi-Squared Equation (****Google, 2023****)*

A table of numbers with numbers

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*Figure 45. The Chi-Squared Distribution Table (****University of Queensland, 2023****)*

The equation to determine the V value, seen in figure 44, was produced in R manually following the method seen in *The Art of Computer Programming Volume 2: Semi-numerical algorithms* (**Knuth, 1998**).

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*Figure 46. A screenshot of the Chi-Squared equation using C# Dice Simulation data*

Figure 46 shows the implementation of the Chi-Squared equation into R using dice simulation data. The *CDice1* list contains the frequency of observed values (Yn) produced by the first implementation of the C# dice simulation while the expected values (np) of each dice face occurring is set to 83.333… (Total Iterations (500) \* Probability of Outcome (1/6)). Every outcome in each dataset has its observed value compared against its expected value producing a V value for each dataset. Eight V values were produced from the collected dice data, two from C#, three from Python, one from JavaScript, one from Random.org, and one from physical dice.

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*Figure 47. A Scatterplot Showing the Frequency of Dice Outcomes*

The frequency of observed values for each of the dice datasets is shown in figure 47. The most immediately noticeable trend is given by the JavaScript dataset which features a noticeably lower frequency of 1s and 6s but the largest frequency of 2s,3s,4s and 5s with all these outcomes occurring significantly more than expected. The remaining datasets all followed a similar trend, with the observed frequency of all possible outcomes occurring between 65 and 95 times.